THE FLUCTUATING WALL PRESSURES UNDER A SEPARATED SUPERSONIC FLOW*

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ABSTRACT

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Measurements have been obtained of the pressure fluctuations under the turbulent, separated region ahead of a forward-facing step at Mach numbers of 3.01 and 4.54. These pressures are significantly larger than the pressures produced by an attached boundary layer. The data can be interpreted as showing that the pressure fluctuations originate from two distinct causes; fluctuations due to changes in the geometry of the separated region and fluctuations due to the turbulent free shear layer. The levels to be expected from each cause can be estimated from a simple model.

Author

INTRODUCTION

This paper presents some measurements of the fluctuating pressure field associated with a separated, supersonic, turbulent boundary layer. The nonsteady forces associated with turbulent flows have become of increasing interest in recent years owing to the fact that flight vehicles are operated in flow regimes with large dynamic pressures and are subject, therefore, to large fluctuating forces. The pressure fluctuations produced by attached turbulent boundary layers have been studied extensively and are understood well enough that reasonable estimates of their levels can be made. It is also important, however, to understand the nonsteady forces caused by separated flows since most vehicles have separated flows over at least part of their boundary. These separated regions exist for a variety of causes;

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e.g., deflected control surfaces or a shock wave impinging on the surface. Our understanding of separated flows is not sufficient for estimating the fluctuating forces associated with them, not only because of the ever-present problem of the turbulence structure but also because the mean flow itself is not understood.

This lack of understanding, as well as the fact that separated regions are produced by a great variety of boundary conditions, makes it very difficult to select a particular example for study that will yield results of some universality. The separated flow studied in this paper, the flow ahead of a forward facing step in a supersonic stream, was selected for experimental convenience and for the fact that extensive studies have been made of the mean flow field.[1, 2] Only sufficient mean data had to be taken to establish that the flow was basically the same as that studied by others, and then the investigation could be limited to the nonsteady features of the flow. In both Refs. 1 and 2 it is stated that the flow is unsteady, but no quantitative measurements of this component of the flow were obtained.

I. EXPERIMENTAL ARRANGEMENT

The flow investigated was the side wall boundary layer of the JPL 20-in. supersonic wind tunnel; the separation was produced with a step obstruction on the wall. Pressure instrumentation was available for obtaining the fluctuating component of the pressure on the wall with a uniform frequency response out to 200 kc. The sensitive element was a piezoelectric disk 0.1 in. thick and 0.1 in.dia., suitably mounted for dominant sensitivity to pressure on its surface.[3] The pressure transducer location was fixed, and the wall pressure distribution on the center line of the separated region was obtained by moving the step along the wall in the stream direction. The mean static pressure on the wall was measured with a Statham gauge at the same streamwise location as the pressure transducer. The movable

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[1] Chapman, D.R., Kuehn, D.M., and Larson, H.K., NACA Rep. 1356, 1958.

[2] Bogdonoff, S.M., Heat Transfer and Fluid Mechanics Institute, Preprints of Papers, University of California at Los Angeles, 1955.

[3] Kistler, A.L., and Chen, W.S., Journal of Fluid Mechanics, Vol. 16, May 1963.

step was a piece of angle iron two in. high and ten in. wide. The bearing surface between the step and the tunnel wall was sealed with a teflon strip so that no leakage occurred under the step.

Measurements were made for freestream Mach numbers of 3.01 and 4.54. The total pressure was 160 cm-Hg in both cases, and the total temperature was about 100° F. The boundary layer thickness ahead of the separated region was 1.50 in. for the M = 3.01 flow, and 2.05 in. for the M = 4.54 flow. The history of these boundary layers was such that they were approximately equilibrium, flat plate, turbulent boundary layers at the separation point. There was no significant heat transfer to the flow.

II. RESULTS

A. The Mean Pressure on the Wall

The mean static pressure on the wall as a function of the distance from the face of the step is shown in Fig. 1 for both experimental cases. The mechanical limits of the step motion prevented any measurements for $x > 10$ in. so that the pressure near the separation point for the M = 4.54 flow could not be obtained. x is the distance from the step face. For the geometry used here, the pressure distribution is a function of the Mach number, the boundary layer Reynolds number, the ratio of the step height h to the boundary layer thickness δ, and the end conditions for the step. The step used here spanned half the tunnel width and was five times as wide as it was high. Chapman, Kuehn, and Larson obtained data for very large ratios of step width to height, with ratios of h/δ approximately the same as those presented here. The boundary layer thicknesses were not included in their report but were obtained from Kuehn,[4] where essentially the same model was used. Bogdonoff has obtained data for a step of about the same aspect ratio as the one used in this report, but his step spanned the tunnel so that the end conditions are not the same as those encountered here. A comparison of the mean pressure distributions for similar values of the relevant parameters is shown in Fig. 2. P_1 is the pressure upstream of the separation point. The three cases show the known features of the mean

[4] Kuehn, D.M., NASA Memo. 1-21-59A, February 1959.

pressure distribution ahead of a forward facing step. The pressure rises rapidly near the separation point and then more slowly as the first maximum is approached. After this peak, p_p, the pressure dips a little and then rises again immediately adjacent to the step face. The results obtained by Chapman, et al, are quite similar to the results obtained here, particularly with respect to the pressure gradient near separation. Bogdonoff's data show a different shape in this region, a result most probably attributable to the effects of the tunnel side wall boundary layers.

The data obtained here have a shape near separation that displays a pressure gradient near the maximum of that obtained by others.

B. The Fluctuating Pressure

Three types of measurements were made of the fluctuating pressures at the wall. The mean square fluctuation level was obtained for both the M = 3.01 and the M = 4.54 cases. The power spectra of the M = 3.01 fluctuations were measured at several points within the separated region. Finally, the space-time correlations of the wall pressure fluctuations were measured for the M = 3.01 flow. Near the separation point, i.e., in the region where the mean pressure gradient is a maximum, the qualitative features of the pressure fluctuations were observed to differ from the features well within the separated region. Away from the separation point, the time history of the pressure at a point, as observed on an oscilloscope, appeared as a normal turbulence signal (i.e., as a finite band width white noise). Near the separation point, the signal showed a distinct on-off character. In this region the signal could best be described as a white noise superimposed on a random square wave whose amplitude was larger than that of the noise. The frequency of the square wave was considerably smaller than 1 kc, and by removing the frequency components below 1 kc with a high-pass filter, the signal was converted to an appearance similar to a white noise with a high percentage square wave modulation. This showed that the high-frequency component of the signal had different amplitudes on the two levels of the square wave.

Measurements of the mean square pressure fluctuation levels were obtained both with and without the square wave filtered out for the M = 3.01 flow. For the M = 4.54 flow, measurements were made only with the square wave filtered out.

These data are shown in Figs. 3 and 4 in two different ways. In Fig. 3 the data are shown as the ratio of the rms pressure level p' to the level p'_1 of the unseparated boundary layer ahead of the separated region. In Fig. 4 the data are compared to the mean pressure $\bar{p}$ to show the actual range of pressure variation at a particular point in the flow. The rms pressure fluctuation level for the attached boundary layer is about 5τ at these Mach numbers[3], where τ is the local mean wall shear. The skin friction coefficient ahead of the separated region is estimated to be about 0.001 for both Mach numbers. Consequently $p'_1/p_1 = 0.031$ for $M = 3.01$, and $p'/p_1 = 0.072$ for $M = 4.54$.

Spectra were measured in the $M = 3.01$ separated flow at $x = 1.55$ in., $x = 6.06$ in., and also ahead of the separated region. These spectra are shown in Fig. 5 plotted against a nondimensional frequency. In order to make the relative shapes more apparent, the ordinate is constructed so that the areas under the spectra are equal. To get the actual power spectra of the pressure, these spectra should be normalized and then multiplied by the value of p'^2 corresponding to the particular location. It is apparent from these spectral shapes that there is relatively more energy at the low frequencies in the separated region than in the attached boundary layer. The various spectra are not similar (affinely related). No spectra were obtained near the separation point since here a significant fraction of the energy was below 1 kc, and the large fluctuations in the spectra at these low frequencies made it difficult to obtain quantitative measurements. The measured spectra show that well within the separated region there is not much energy below 1 kc ($f\delta/u = 0.06$). This fact is also borne out by the p' measurements presented earlier, since well within the separated region, no measurable difference occurred when the signal was cut off below 1 kc.

If $p(x,y,t)$ is the instantaneous pressure fluctuation at the position x, y, then the space-time correlation measured here is defined as

$$\langle p^2 \rangle R(\Delta x, \Delta t) = \langle p(x + \Delta x, y, t + \Delta t) p(x, y, t) \rangle$$

where $\langle\,\rangle$ denotes a time average. x is in the stream direction. Space-time correlations were measured by two pressure transducers separated by 1/2 in.

The results of these measurements are shown in Fig. 6 for two different locations within the M = 3.01 separated flow. Following the practice for attached boundary layer measurements, a convection U_c is defined as the quotient of the transducer separation Δx and the time delay at the maximum of the correlation. This velocity is also shown in Fig. 6. It is known that the convection speed for the attached boundary layer at this Mach number is 0.6 of the free stream speed[3]. This value is also shown in the figure. The remarkable thing about these results is that the convection speed for the separated flow, which can be roughly described as the speed of the turbulent eddies most efficacious in producing the wall pressure, is in the direction of the external stream. Since the flow at the wall in the separated region is on the average in the opposite direction, the pressure fluctuations move in a direction opposite the local mean velocity.

III. DISCUSSION

A. The Pressure Near the Separation Point

The measurements of the mean and fluctuating components of the wall pressure in the neighborhood of the separation point presented in the preceding section can be combined into one consistent picture. The most striking feature of the measurements near separation was the qualitative behavior of the signal. A sketch of this signal is shown in Fig. 7. The pressure jumps back and forth between the levels p_1 and p_2, and at each level the pressure oscillates with an amplitude characteristic of that level. The simplest explanation for such a behavior is that at any instant the pressure distribution along the wall is a step function, with the lower value p_1 in the region ahead of the separation point and p_2 in the separated region. If the location of this jump is not stationary but moves about over a restricted range, the pressure at some point in this range would have the general features that were observed. The measurements are explained quantitatively by the calculations below. The various pressure levels are defined in Fig. 8. p_1 and p_2 are the mean pressures on the bottom and top of the step (i.e., the means computed only on that set of times when the high or low pressure region of the step is present). The instantaneous fluctuations about these means are denoted by p_1' and p_2', respectively. ϵ is defined as the fraction of time that the high pressure region is over the point of interest;

therefore, $(1 - \epsilon)$ is the fraction of time that the low pressure region is over this point. It is assumed that p'_1 and p'_2 are uncorrelated with each other. It then follows that:

The mean static pressure at the point of interest is

$$p = \epsilon p_2 + (1 - \epsilon)p_1 \qquad (1)$$

The mean square fluctuation around the mean pressure is

$$p'^2 = \epsilon(1 - \epsilon)(p_2 - p_1)^2 + \epsilon p_2'^2 + (1 - \epsilon)p_1'^2 \qquad (2)$$

The mean square pressure level when the low frequency step is filtered out is

$$p'^2 \text{ (filtered)} = \epsilon p_2'^2 + (1 - \epsilon)p_1'^2 \qquad (3)$$

Therefore, ϵ can be obtained from the mean pressure measurements, i.e.,

$$\epsilon = \frac{(p/p_1 - 1)}{(p_2/p_1 - 1)} \qquad (4)$$

and the other quantities can be predicted by the relations given above.

A p_2 was selected that gave the best fit to the computed p'^2 and and p'_2 was selected corresponding to this value of p_2. This value of p_2 was seen to occur at the location of an inflection point in the mean pressure curve. This second inflection occurs between the steeply rising portion of the curve and the hump containing the peak pressure. This second inflection is present in most of the measurements of the mean pressure distributions for a step flow (i.e., in the data of Chapman, Kuehn and Larson). Since the flow studied by them was reasonably two dimensional owing to the large aspect ratio of their model, the relationship between flow deflection and pressure change should be given by two dimensional theory. If it is assumed that the external flow is deflected through an angle determined by the line between the separation point (the first inflection) and the corner of the step, then the computed pressure behind the shock wave associated with this deflection is roughly that at the second inflection of their data.

The results of the calculation of the low frequency fluctuation levels on the basis of the mean measurements are shown in Fig. 8. It is seen that there is good quantitative agreement between the measured and computed results. The small differences can most probably be attributed to the fact that the step function does not have precisely square corners.

If this low frequency unsteadiness of the pressure near the mean separation point is caused by motion of the instantaneous location of the separation point, as the data certainly suggest, then the question arises as to what causes this motion. One possibility is that it is the result of an acoustic oscillation of the entire separated region as is sometimes observed in cavity flows[5]. However, two pieces of evidence discredit this explanation. One would expect such an oscillation to be reasonably periodic and to be detectable at all points within the separated region. The spectra obtained within the separated region, however, showed no evidence of a strong low frequency (less than 1 kc) energy concentration. An acoustic oscillation would also be expected to be reasonably well organized across the width of the separated region, mainly because of the reflection from the flat face of the step. Schlieren pictures taken parallel to the separation line (z-direction), however, do not show any organized pattern within the separated region. It seems likely therefore, that an acoustic mechanism is not the dominant cause of the separation point motion.

Another possible explanation for this motion is that the dividing surface is randomly distorted in the z-direction. This conjecture is supported by the fact that the motion of the separation point is not observed in either still or motion pictures of the flow. The extent of this region (almost the entire region of the steep pressure rise) is big enough compared to the boundary layer thickness that it certainly should be visible if the motion were uniform over the width of the separated region. A random variation in the z-direction of the separation point, however, would be averaged in a picture normal to the flow. Furthermore, a cross stream distortion of the flow is

[5] Krishnamurty, K., NACA Technical Note 3487, 1955.

a plausible explanation of the observed effect since for this supersonic flow any perturbation that would increase the deflection angle of the dividing surface would increase the local pressure, push aside the slowly moving, low density recirculating fluid and increase the perturbation. Measurements by Kuehn[4] have shown that a supersonic turbulent boundary layer can support a larger pressure rise without separating than that encountered for the step flow, so that some motion of the separation point is possible without contradicting his measurements. The motion of the separation point is limited because if the angle gets too large, either a new separation bubble is formed ahead of the old one or some other mechanism intervenes.

B. The Pressure Inside the Separated Region

The wallpressures inside the separated region and away from the separation point seem to be caused by the turbulent activity in the free shear layer near the dividing stream line. The fact that the convection speed is in the external flow direction and is of about the same size as that for an attached boundary layer clearly implies this. The nonsimilarity of the spectra shows that other mechanisms are also operating in certain regions, but they appear to be secondary influences on the wall pressure compared to the free shear layer turbulence.

The pressure fluctuations on the wall are related to the forces necessary to balance the momentum directed perpendicular to the wall in the turbulent motion. For an attached boundary layer, these forces are proportional to the mean turbulent shear. The pressures occurring for the separated layer are likewise related to the normal turbulent momentum and most probably are proportional to the shear on the free shear layer. The proportionality factor, however, depends on all the variables characterizing the flow since the presence of the recirculating flow modifies the pressure distribution on the wall necessary to produce a given force. The geometry of this region as well as the dynamical properties of the flow depend on M, h/δ, etc. The level of the shear itself in the free shear layer must depend somewhat on these variables.

The assumption that the pressure fluctuation is related to the shear in the same way as for an attached boundary layer leads to an estimate of p' between 0.05 and 0.1 of the mean dynamic pressure, q. The incompressible value for the shear coefficient on the dividing line of a fully developed free shear layer is in the range from 0.01 to 0.02. This value decreases slowly with M according to most investigators. For the measurements here where $Cf = 0.001$, p'/p_1' is between 10 and 20, which is in the range of the measurements. Since for the flow considered here, it is unlikely that the shear layer is fully developed, the actual shear coefficient probably lies between that for an attached boundary layer and the range stated above. The buffer of recirculating air (between the active turbulent region and the wall which permits partial cancellation of adjacent plus and minus pressure fluctuations) would also be expected to decrease the pressure fluctuation level at the wall compared to that which would be obtained if this "dead air" were not present. Perhaps the fact that the convection velocity is about equal to the sound velocity throughout the recirculating region and at the wall inhibits the cancellation effect at these high Mach numbers.

If the interior fluctuations are produced mainly by the turbulence in the shear layer, one would expect to encounter high levels also in wake regions. The only available measurements of this case are those of Eldred[6] who made measurements at subsonic speeds behind a bluff body. His results showed only a small increase of the pressure levels due to separation over those for an attached layer.

One possible explanation for this result within the framework sketched above is that for subsonic flows, the dead air space does decrease the wall pressures by permitting the pressure field produced by a given momentum change to spread, or, alternatively, by allowing adjacent high and low pressure producing regions to partially cancel.

In order to construct a theory for the transport properties across a separated region, it is necessary to have some model for the entire flow. As one part of this model, it is necessary to make assumptions about the properties of the recirculating flow region. It is useful, therefore, to examine the relative size of the pressure fluctuations in the recirculating region compared to the mean dynamic pressure of the recirculating flow in order to establish whether a steady model for this region is a good approximation.

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[6] Eldred, K. McK., Journal of Acoustic Society of America, Vol. 33, January 1961.

Measurements of the reverse flow are difficult to obtain, but some rough measurements by the author and by others indicate that the maximum reverse flow velocity is in the range of from 0.2 to 0.4 of the velocity outside of the free shear layer. For the approximately adiabatic flow considered here, this implies that the internal flow is subsonic with respect to the boundary and that its static temperature is approximately equal to the free stream total temperature. This assumption has some experimental backing and is consistent with the assumption that the turbulent Prandtl number is near one. If the internal velocity is denoted by U_i, the external velocity by U_o, and the internal and external densities by ρ_i, and ρ_o, then the internal dynamic pressure is given by

$$q_i = \frac{\rho_i}{\rho_o}\left(\frac{U_i}{U_o}\right)^2 \frac{1}{2}\rho_o U_o^2 \tag{5}$$

U_o and ρ_o are calculated for the condition behind the shock wave that appears near the separation point. The mean pressure inside the separation region is assumed near the peak pressure, p_p, defined earlier. The distribution of the pressure fluctuation levels, excluding the region near the separation point, has a shape similar to that of the mean pressure. The location of the peak is not the same, however. If the peak value is taken as characteristic of these levels, the experimental results give $p'/p_1' = 19$ for $M = 3.01$, and $p'/p_1' = 12.8$ for $M = 4.54$. A tentative calculation shows that if U_i/U_o is taken as 0.375 for the $M = 3.01$ flow, q_i is equal to the peak pressure fluctuation level. Using this value of U_i/U_o to compute the value of the peak fluctuation level at $M = 4.54$, it is found that $p'/p_1' = 12.3$, which is close to the measured value This calculation shows that the fluctuations in pressure within the separated region are of the same order as the mean dynamic pressure within the separated region. Such a large value for p' compared to q_i indicates that the internal flow can change direction, and that the flow is sufficiently agitated that the nonsteadiness should be included in any heat transfer or other transport calculation for the separated flow.

A question raised by the present results is: What is the form of the pressure distribution as the step height-to-boundary layer thickness ratio goes to infinity? If the distribution is controlled by the motion of the separation point, which in turn is related to some gross features of the whole flow, the pressure distribution near separation would be expected to scale with h as h/δ goes to infinity. Alternatively, if the distribution is a so-called free interaction, it would scale with δ as h/δ goes to infinity. Existing published data do not cover a sufficient range of h/δ to give an answer to this question.

Of practical importance is the question whether anything can be done to limit the force fluctuations caused by a separated flow at supersonic speeds. If the conjectures concerning the low frequency buffeting are true, it might be possible to minimize this component of the force by fixing the location of the separation line. That is, it might be possible to "trip" the separation by a small ramp so that the high pressure region has a fixed area and, consequently, does not contribute an additional fluctuating force to the vehicle by moving about. The fluctuating pressures occurring well within the separated region seem to arise from the combined action of the turbulent shear layer and the recirculating flow, so that it is unlikely that there exists a method for drastically modifying these levels other than by avoiding the separation itself.

IV. CONCLUSIONS

Associated with a separated region on a vehicle in a supersonic stream are large time dependent forces. On the basis of data presented here, these forces can be resolved into two components; a low-frequency buffeting caused by changes in the geometry of the separated region, and a wide band fluctuation apparently originating in the free shear layer of the separated region. The magnitude of the loading produced by each component can be estimated on the basis of a plausible analysis presented here, but it is clear that much more experimental work is required before any reliable calculation scheme can be constructed.

FIGURES

1. Mean pressure distributions for the two experimental cases
2. Comparison of mean pressure distributions for approximately equal values of M and h/δ
3. The rms pressure fluctuation levels
4. The pressure fluctuation levels compared to the mean pressures
5. Shapes of the power spectra of the pressure fluctuations (the areas under the spectra are equal)
6. Space-time correlations and the convection speed within the separated region
7. Sketch of the time variation of the pressure at various points of the separated region
8. The pressure distribution near the separation point

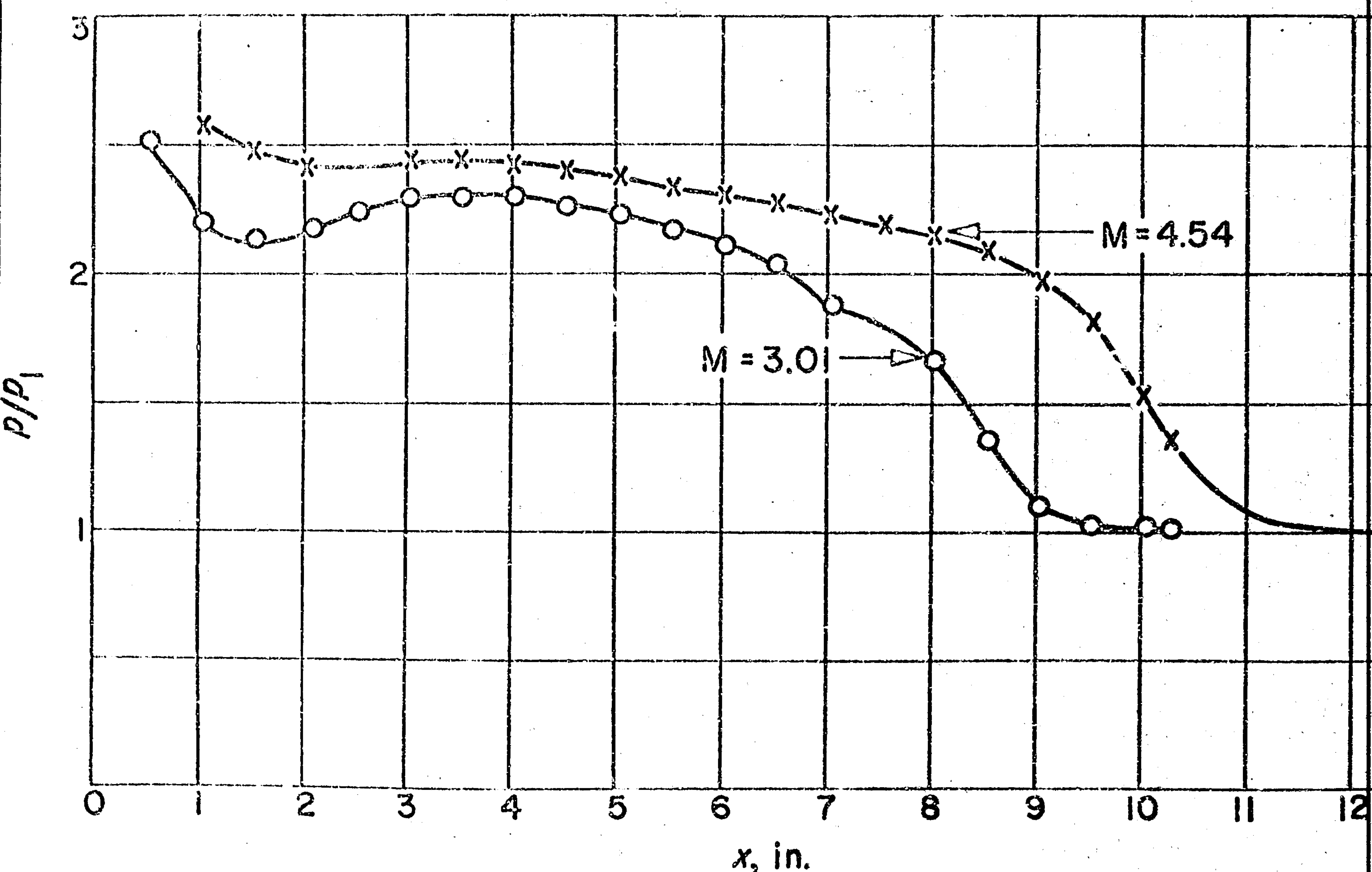
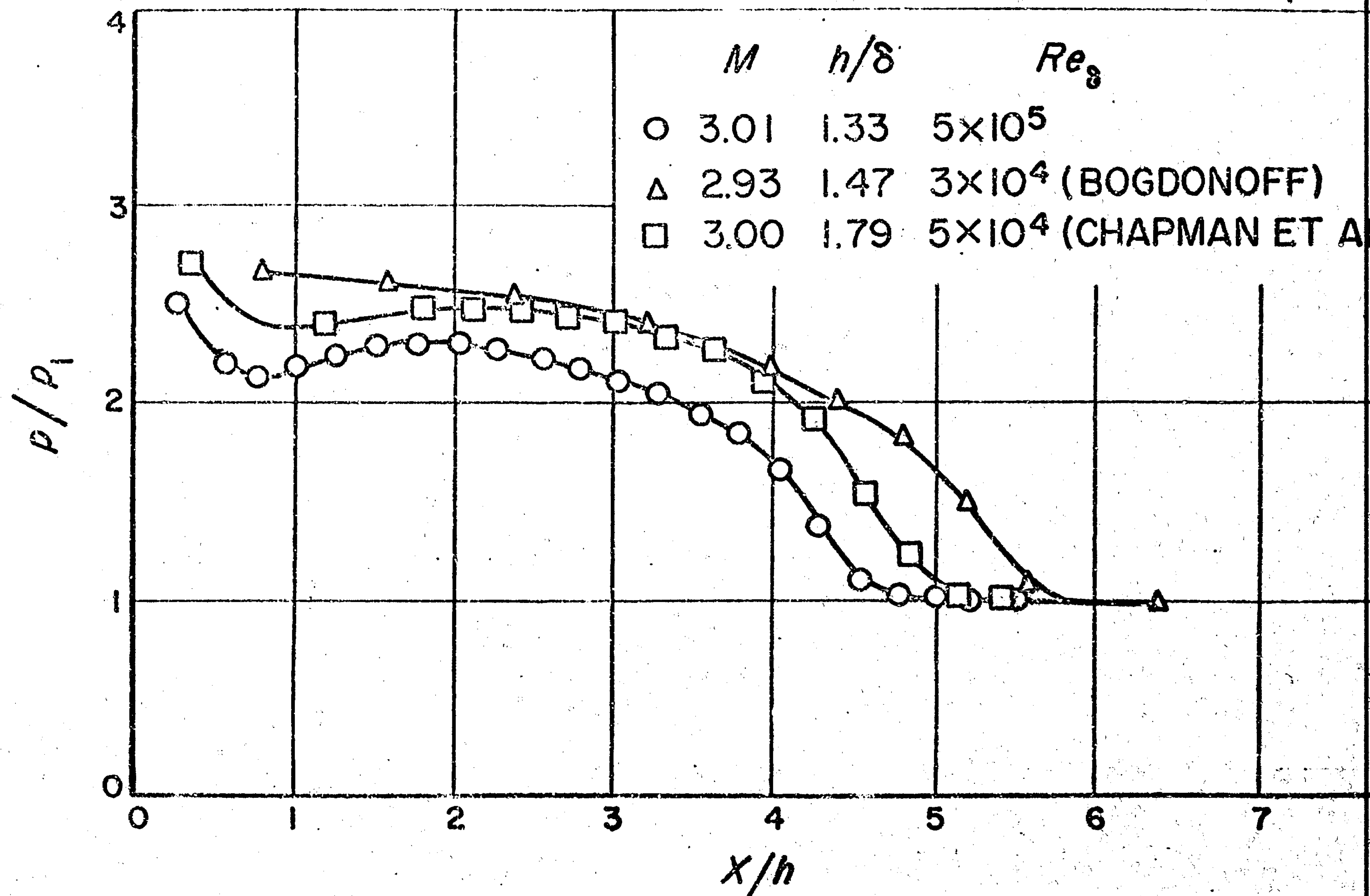
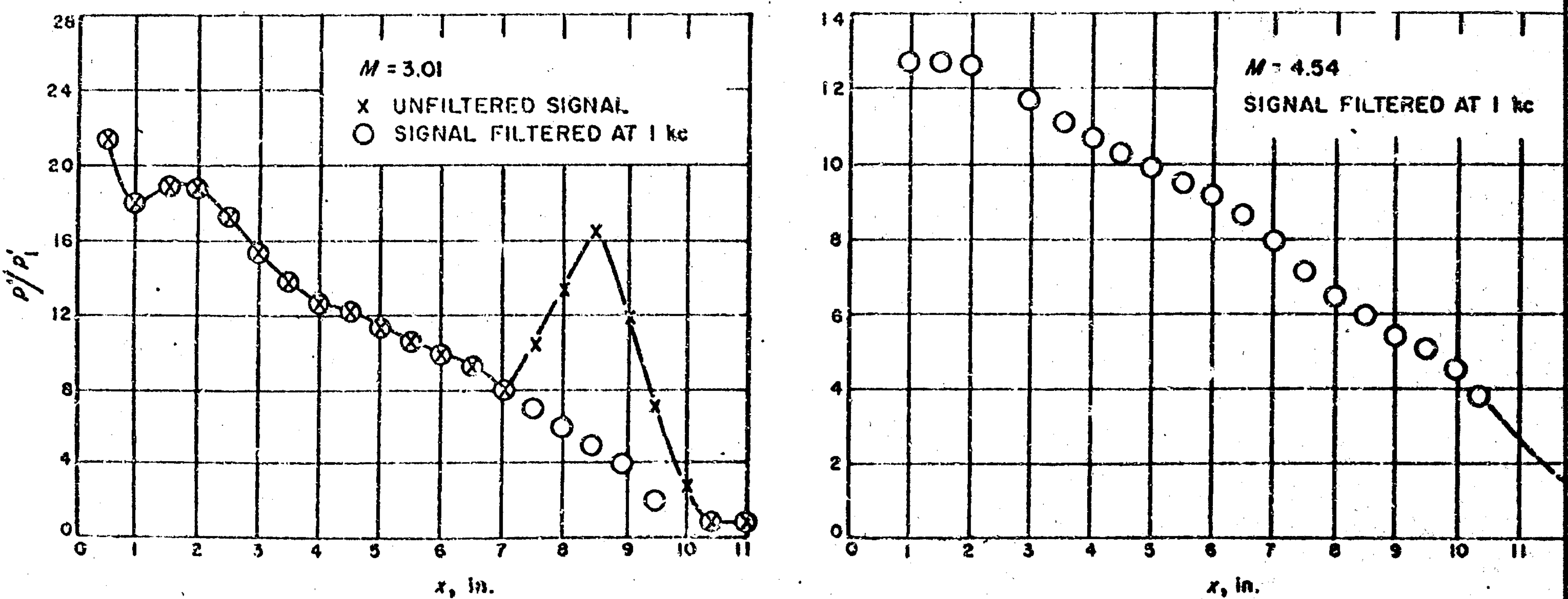


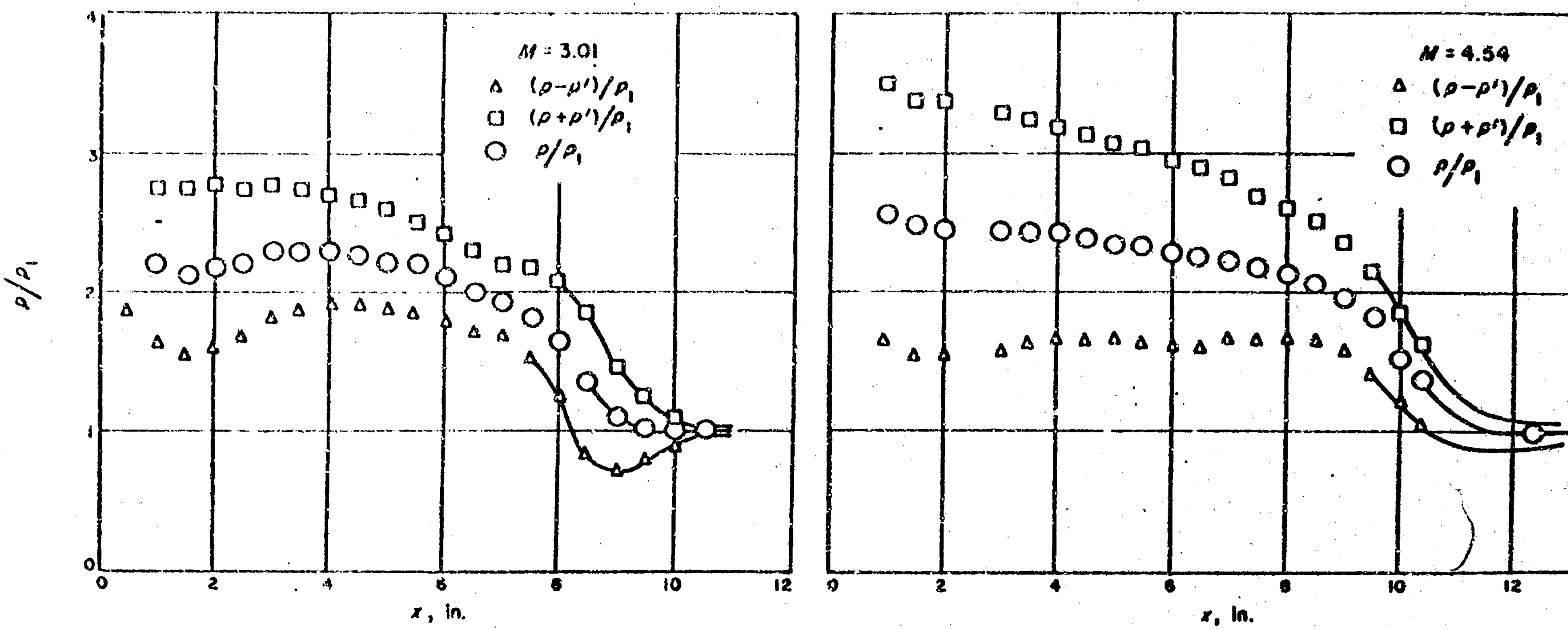
Fig. 1


M h/δ Re_δ
3.01 1.33 5×10^5
2.93 1.47 3×10^4 (BOGDONOFF)
3.00 1.79 5×10^4 (CHAPMAN ET A
p/p_1
X/h
0
1
2
3
4
5
6
7



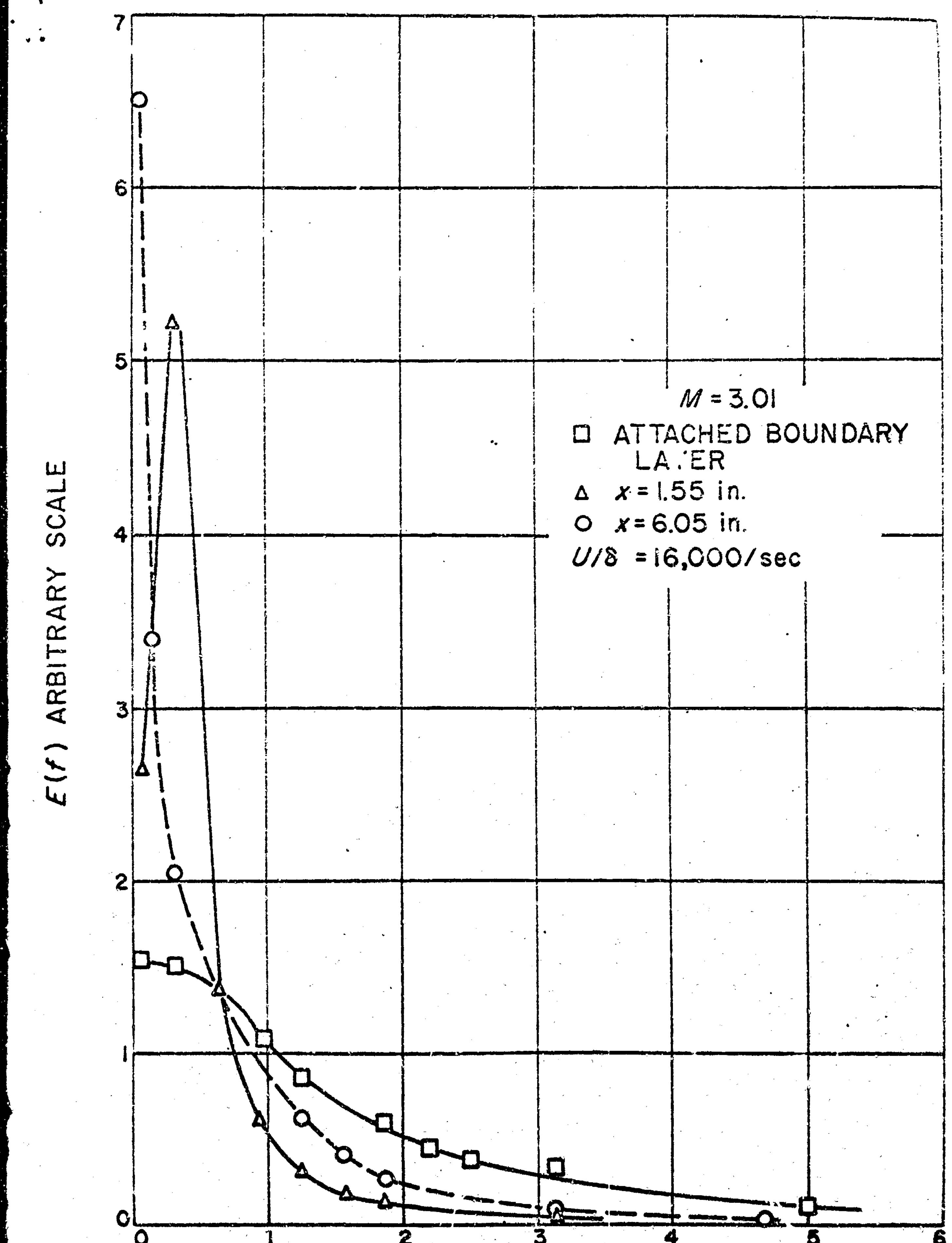
$M = 3.01$
x UNFILTERED SIGNAL
O SIGNAL FILTERED AT 1 kc
ρ'/ρ'_1
x, in.
$M = 4.54$
SIGNAL FILTERED AT 1 kc
x, in.

Fig 3



$M = 3.01$
$(\rho - \rho')/\rho_1$
$(\rho + \rho')/\rho_1$
ρ/ρ_1
$M = 4.54$
$(\rho - \rho')/\rho_1$
$(\rho + \rho')/\rho_1$
ρ/ρ_1
ρ/ρ_1
0
1
2
3
4
0
2
4
6
8
10
12
x, in.
x, in.

Fig. 4



M = 3.01
ATTACHED BOUNDARY LA:ER
x = 1.55 in.
x = 6.05 in.
U/δ = 16,000/sec
E(f) ARBITRARY SCALE
0
1
2
3
4
5
6
7

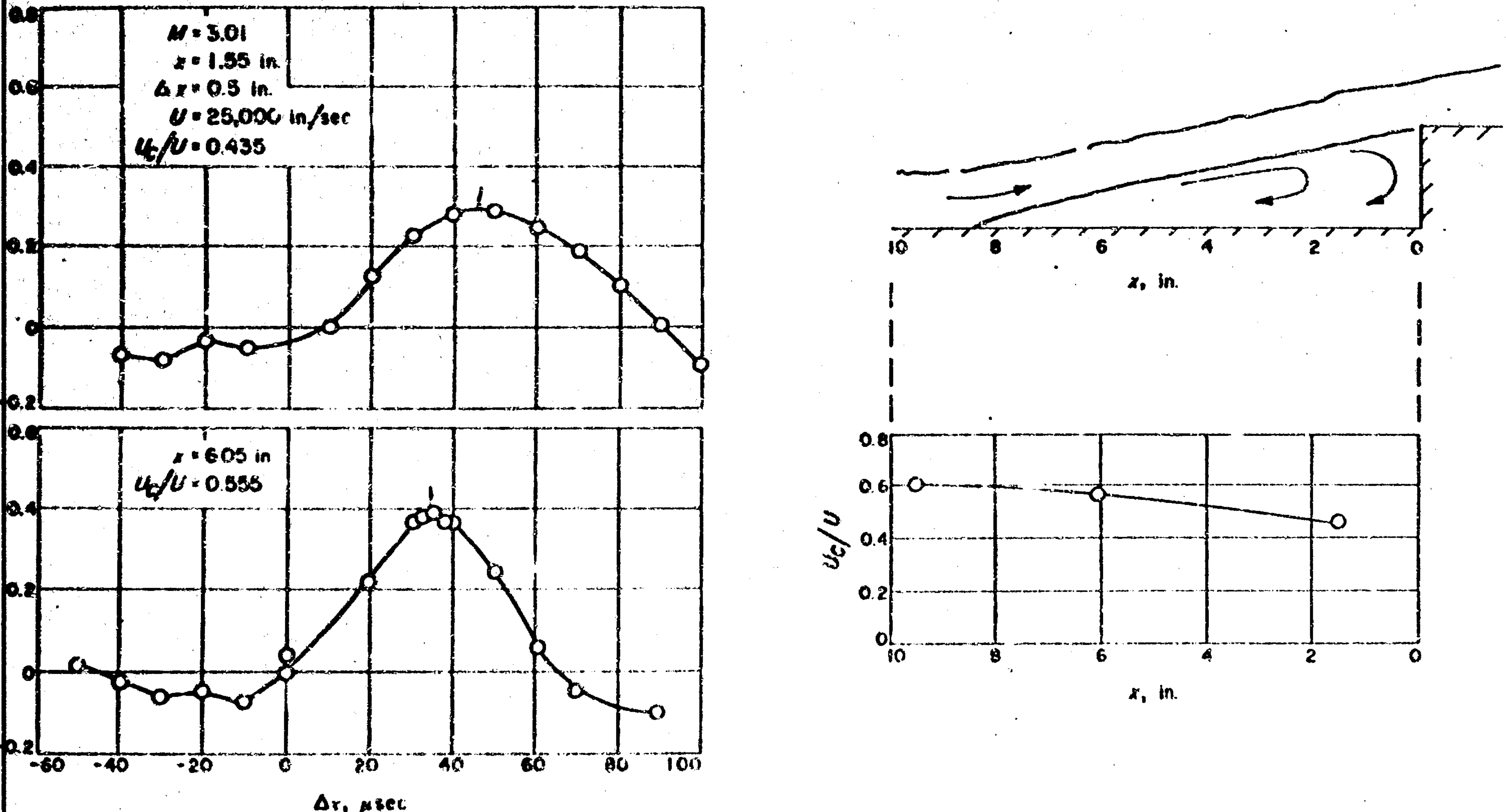

M = 3.01
x = 1.55 in.
Δx = 0.5 in.
U = 25,000 in/sec
u_c/U = 0.435
x = 6.05 in
u_c/U = 0.555
Δτ, μsec
-60 -40 -20 0 20 40 60 80 100
x, in.
10 8 6 4 2 0
u_c/U
0 0.2 0.4 0.6 0.8

Fig. 6

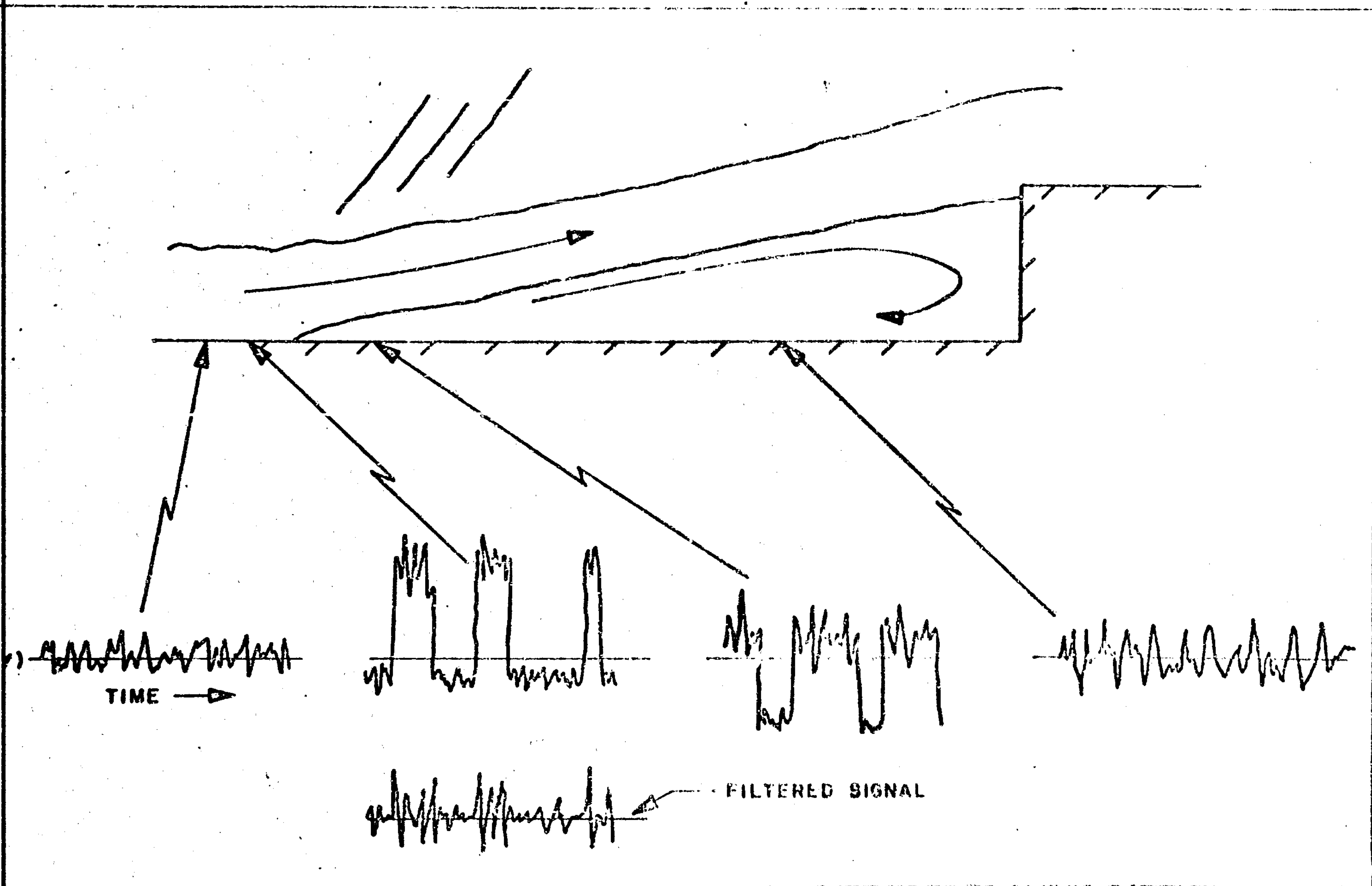
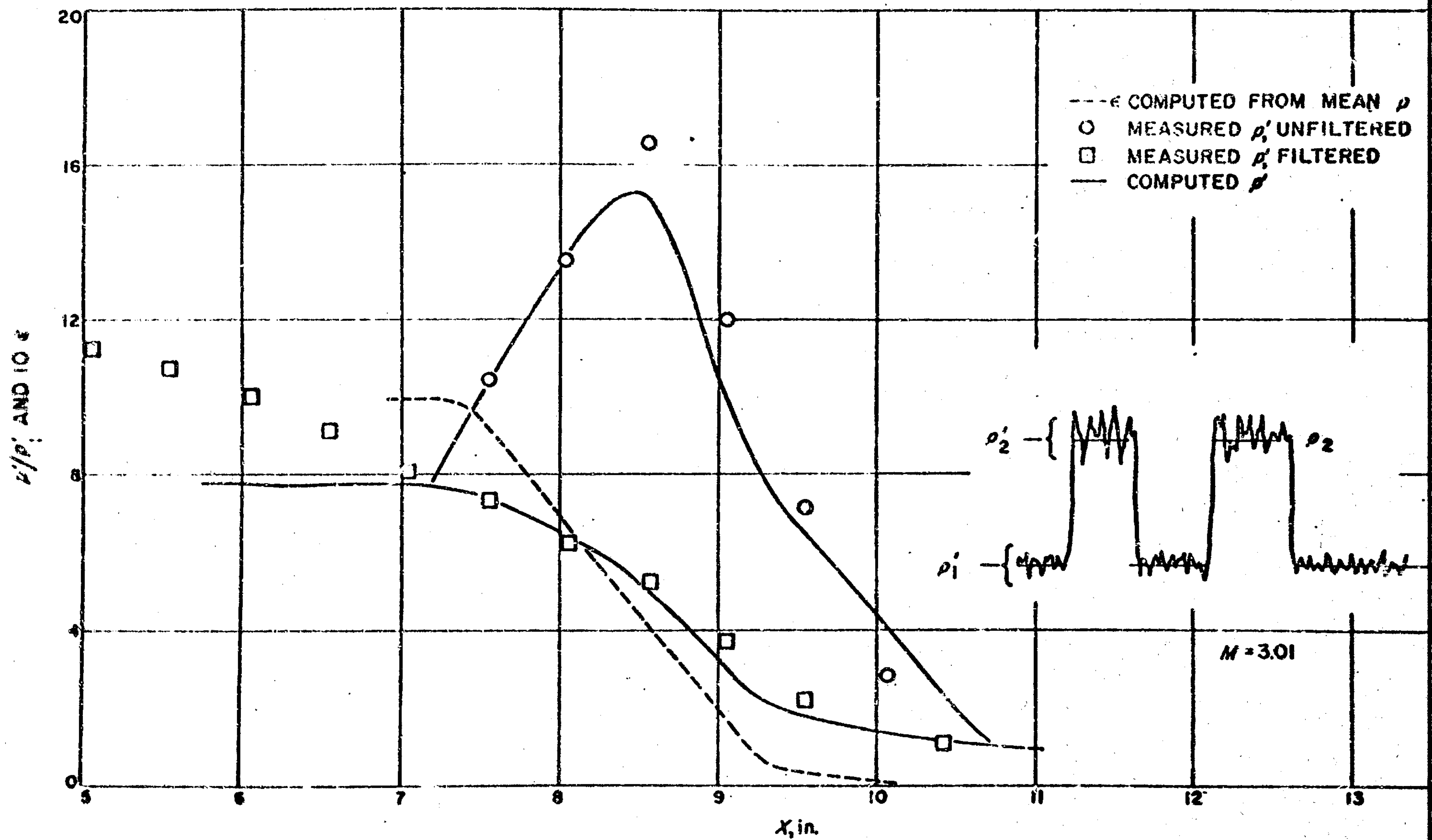


Fig. 7



- - - ε COMPUTED FROM MEAN ρ
○ MEASURED ρ′ UNFILTERED
□ MEASURED ρ′ FILTERED
— COMPUTED ρ′
ρ′₂
ρ₂
ρ′₁
M = 3.01
ρ′/ρ′ AND 10 ε
X, in.
20
16
12
8
4
0
5
6
7
8
9
10
11
12
13

Fig. 8